Chap4 Boolean Algebra and Logic Simplification

- Laws of Boolean Algebra
  - **Commutative Laws**
  - **Associative Laws**
  - **Distributive Law**
  - Rules of Boolean Algebra
- DeMorgan's Theorems
- Logic Simplification Using Boolean Algebra
- Standard Forms of Boolean Expressions
  - The Sum-of-Products (SOP) Form
  - The Standard SOP Form
  - The Product-of-Sums (POS) Form
  - The Standard POS Form
  - Converting SOP Expressions to Truth Table Format
  - Converting POS Expressions to Truth Table Format
  - Determining Standard Expressions from a Truth Table
- The Karnaugh Map
  - The 3-Variable Karnaugh Map
  - The 4-Variable Karnaugh Map
  - Cell Adjacency
- Karnaugh Map SOP Minimization
  - Mapping a Nonstandard SOP Expression
  - Karnaugh Map Simplification of SOP Expressions
  - Mapping Directly from a Truth Table
  - "Don't Care" Conditions
- Karnaugh Map POS Minimization
  - Mapping a Standard POS Expression
  - Converting Between POS and SOP Using the Karnaugh Map

解法,

变量:

A { 0, 1

Chap4 Boolean Algebra and Logic Simplification

Laws of Boolean Algebra

Commutative Laws

$$A + B = B + A$$

$$AB = BA$$

0 1

$2^2 = 4$

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

Associative Laws

$$A + (B + C) = (A + B) + C$$

$$A(BC) = (AB)C$$

$A \cdot B$

Distributive Law

$$A(B + C) = AB + AC$$

Rules of Boolean Algebra

12个同学,

TABLE 4–1

Basic rules of Boolean algebra.

1. $A + 0 = A$	**7.** $A \cdot A = A$
2. $A + 1 = 1$	**8.** $A \cdot \overline{A} = 0$
3. $A \cdot 0 = 0$	**9.** $\overline{\overline{A}} = A$
4. $A \cdot 1 = A$	**10.** $A + AB = A$
5. $A + A = A$	**11.** $A + \overline{A}B = A + B$
6. $A + \overline{A} = 1$	**12.** $(A + B)(A + C) = A + BC$

A, *B*, or *C* can represent a single variable or a combination of variables.

AND

1律性

2律

2.2

20, 21

$A = A \cdot 1 + AB = A(1+B) = A \cdot 1 = A$

$(A+\overline{A}B) = A + AB + \overline{A}B$

$= A + (A + \overline{A})B$

$= A + B$

DeMorgan's Theorems

DeMorgan's first theorem is stated as follows:

The complement of a product of variables is equal to the sum of the complements of the variables.

$$\overline{XY} = \overline{X} + \overline{Y}$$

$AB + \overline{AB}C = AB + C.$

$= A + \overline{A}\,\overline{B} = A + B$

DeMorgan's second theorem is stated as follows:

The complement of a sum of variables is equal to the product of the complements of the variables.

$$\overline{X + Y} = \bar{X}\bar{Y}$$

直接

$AA + AC + BA + BC$

$= A + AC + AB + BC.$

$= A + AB + BC$

$= A + BC$ (11강)

EXAMPLE 4–5

Apply DeMorgan's theorems to each of the following expressions:

(a) $\overline{(A + B + C)D}$ $= \overline{A+B+C} + \overline{D} = \overline{A}\,\overline{B}\,\overline{C} + \overline{D}$

(b) $\overline{ABC + DEF}$ $= (\overline{A}+\overline{B}+\overline{C})(\overline{D}+\overline{E}+\overline{F})$

(c) $\overline{A\overline{B} + \overline{C}D + EF}$ $= \overline{A\overline{B}}\;\overline{\overline{C}D}\;\overline{EF} = (\overline{A}+B)(C+\overline{D}) = (\overline{A}C + \overline{A}\overline{D} + BC + B\overline{D})(\overline{E}+\overline{F})$

EXAMPLE 4–6

Apply DeMorgan's theorems to each expression:

(a) $\overline{\overline{(A + B)} + \overline{C}}$ $= ABC$ $= (A+B)C = \overline{\overline{A+B}\;\overline{C}}$

(b) $\overline{(\overline{A} + B) + CD}$ $= \overline{(\overline{A}+B)}\,\overline{CD} = (A\overline{B})\overline{CD}$

(c) $\overline{(A + B)\overline{C}\overline{D} + E + \overline{F}}$

$= \overline{(A+B)\overline{C}\overline{D}}\,(\overline{E+\overline{F}}) = (\overline{A}\overline{B}+C+D)\overline{E}F$

EXAMPLE 4–7

The Boolean expression for an exclusive-OR gate is $A\overline{B} + \overline{A}B$. With this as a starting point, use DeMorgan's theorems and any other rules or laws that are applicable to develop an expression for the exclusive-NOR gate.

SECTION 4–3 CHECKUP

1. Apply DeMorgan's theorems to the following expressions:
   (a) $\overline{ABC} + (\overline{\overline{D} + E})$ **(b)** $\overline{(A + B)C}$ **(c)** $\overline{A + B + C} + \overline{\overline{D}E}$

$A \oplus B = \overline{A}B + A\overline{B}$

$\overline{A \oplus B} = \overline{\overline{A}B + A\overline{B}} = (A+\overline{B})(\overline{A}+B)$

$= A\overline{A} + \overline{A}\,\overline{B} + AB + B\overline{B}$

$= \overline{A}\,\overline{B} + AB$

$\|$

$A \odot B = \overline{A}\,\overline{B} + AB$

Boolean Expression for a Logic Circuit

To derive the Boolean expression for a given combinational logic circuit, begin at the left-most inputs and work toward the final output, writing the expression for each gate. For the example circuit in Figure 4–18, the Boolean expression is determined in the following three steps:

1. The expression for the left-most AND gate with inputs C and D is CD.
2. The output of the left-most AND gate is one of the inputs to the OR gate and B is the other input. Therefore, the expression for the OR gate is $B + CD$.
3. The output of the OR gate is one of the inputs to the right-most AND gate and A is the other input. Therefore, the expression for this AND gate is $A(B + CD)$, which is the final output expression for the entire circuit.

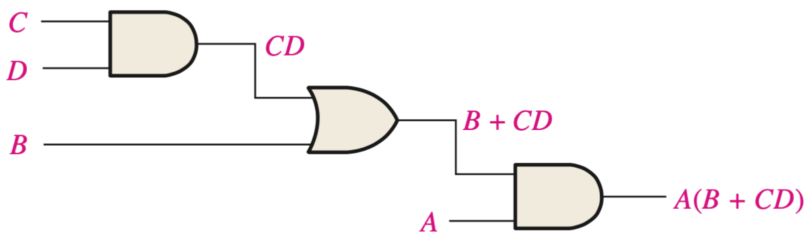


FIGURE 4–18 A combinational logic circuit showing the development of the Boolean expression for the output.

Logic Simplification Using Boolean Algebra

$$AB + A(B + C) + B(B + C)$$

$$[A\bar{B}(C + BD) + \bar{A}\bar{B}]C$$

$$\bar{A}BC + A\bar{B}\bar{C} + \bar{A}\bar{B}\bar{C} + A\bar{B}C + ABC$$

$$\overline{AB + AC} + \bar{A}\bar{B}C$$

$$\text{(a) } A + AB + A\bar{B}C \quad \text{(b) } (\bar{A} + B)C + ABC \quad \text{(c) } A\bar{B}C(BD + CDE) + A\bar{C}$$

Standard Forms of Boolean Expressions

The Sum-of-Products (SOP) Form

$$AB + ABC$$
$$ABC + CDE + \bar{B}C\bar{D}$$
$$\bar{A}B + \bar{A}B\bar{C} + AC$$

The Standard SOP Form

A *standard SOP expression* is one in which *all* the variables in the domain appear in each product term in the expression.

$A\bar{B}CD=1$ $\quad m_{11}$

1011 最低位

$A\cdot\bar{B}\cdot C\cdot D$ 最高位 最小项

$$A\bar{B}CD + \bar{A}\bar{B}C\bar{D} + ABC\bar{D} = m_{11} + m_2 + m_{12}$$

0010

$$= \sum m_i \quad (i = 2, 11, 12)$$

$$=A\bar{B}C(D+\bar{D})$$

$$A\bar{B}C + \bar{A}\bar{B} + AB\bar{C}D$$

$$\bar{A}\bar{B}(C+\bar{C})(D+\bar{D})$$

The Product-of-Sums (POS) Form

$$(\bar{A}+B)(A+\bar{B}+C)$$
$$(\bar{A}+\bar{B}+\bar{C})(C+\bar{D}+E)(\bar{B}+C+D)$$
$$(A+B)(A+\bar{B}+C)(\bar{A}+C)$$

The Standard POS Form

A *standard POS expression* is one in which *all* the variables in the domain appear in each sum term in the expression.

$$(\bar{A}+\bar{B}+\bar{C}+\bar{D})(A+\bar{B}+C+D)(A+B+\bar{C}+D)$$

EXAMPLE 4–17

Convert the following Boolean expression into standard POS form:

$$(A + \bar{B} + C)(\bar{B} + C + \bar{D})(A + \bar{B} + \bar{C} + D)$$

SECTION 4–6 CHECKUP

1. Identify each of the following expressions as SOP, standard SOP, POS, or standard POS:

(a) $AB + \bar{A}BD + \bar{A}C\bar{D}$ **(b)** $(A + \bar{B} + C)(A + B + \bar{C})$

(c) $\bar{A}BC + AB\bar{C}$ **(d)** $(A + \bar{C})(A + B)$

Converting SOP Expressions to Truth Table Format

EXAMPLE 4–20

Develop a truth table for the standard SOP expression $\bar{A}\bar{B}C + A\bar{B}\bar{C} + ABC$.

Converting POS Expressions to Truth Table Format

EXAMPLE 4–21

Determine the truth table for the following standard POS expression:

$$(A + B + C)(A + \overline{B} + C)(A + \overline{B} + \overline{C})(\overline{A} + B + \overline{C})(\overline{A} + \overline{B} + C)$$

Determining Standard Expressions from a Truth Table

EXAMPLE 4–22

From the truth table in Table 4–8, determine the standard SOP expression and the equivalent standard POS expression.

TABLE 4–8

Inputs			Output
A 最高	B	C	X
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1 ✓
1	0	0	1 ✓
1	0	1	0
1	1	0	1 ✓
1	1	1	1 ✓

m_3 $\overline{A}BC$

m_4 $A\overline{B}\overline{C}$

m_6 $AB\overline{C}$

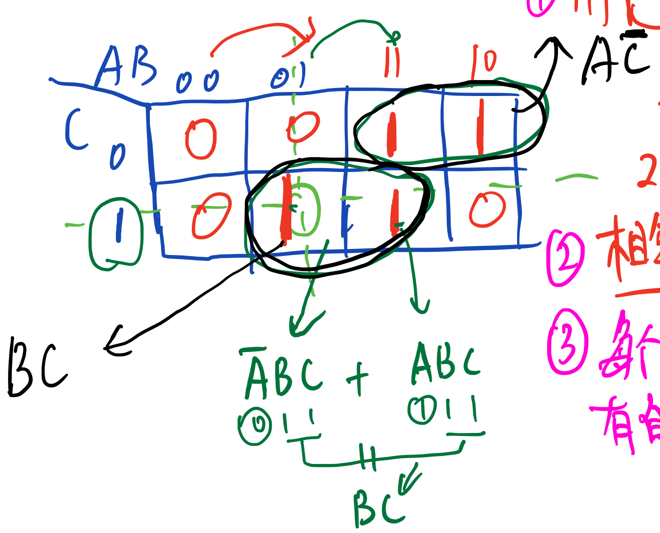
m_7 ABC

$$X = \overline{A}BC + A\overline{B}\overline{C} + AB\overline{C} + ABC = A\overline{C} + BC$$

The Karnaugh Map

$= \Sigma m_i \ (i = 3, 4, 6, 7)$

The 3-Variable Karnaugh Map

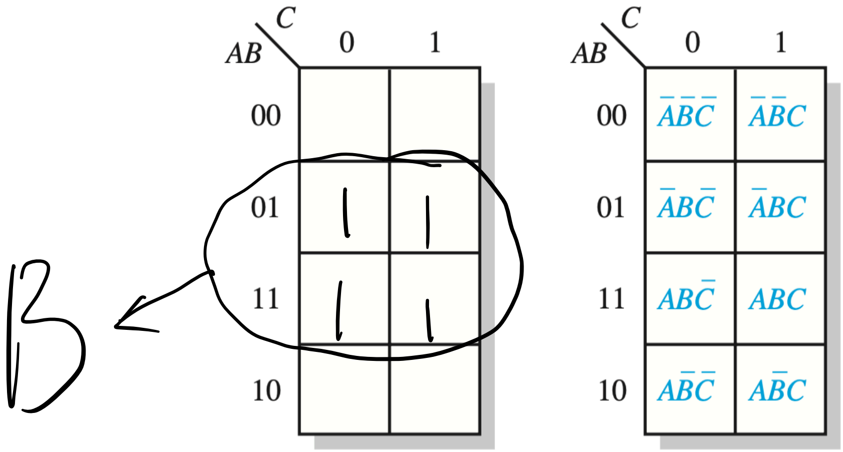


① 用 □ ○ 圈 2^n 个 1。

2、4、8、16

② 相邻的 1

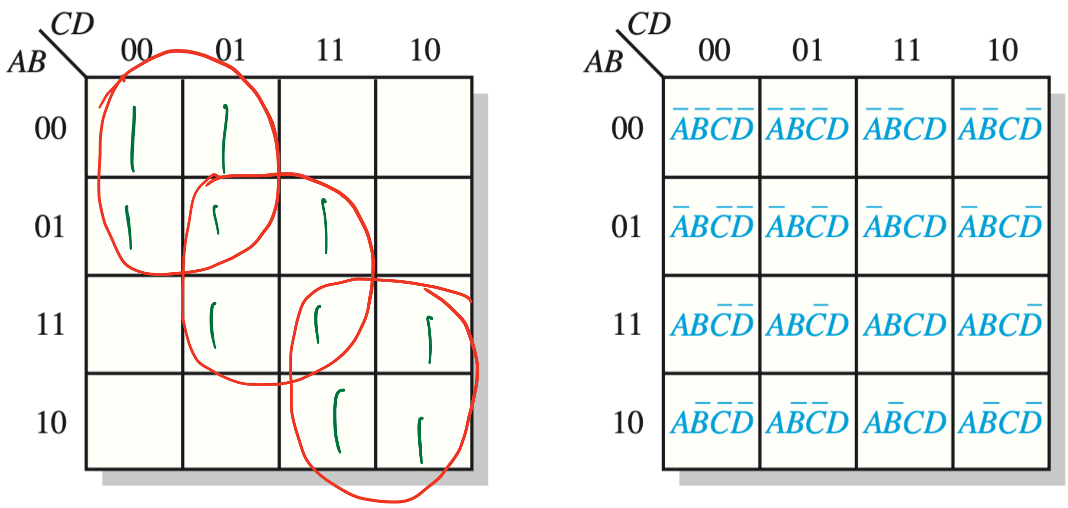
③ 每个○必须有自己的 1。



(a) (b)

FIGURE 4–25 A 3-variable Karnaugh map showing Boolean product terms for each cell.

The 4-Variable Karnaugh Map



(a) (b)

FIGURE 4–26 A 4-variable Karnaugh map.

Cell Adjacency

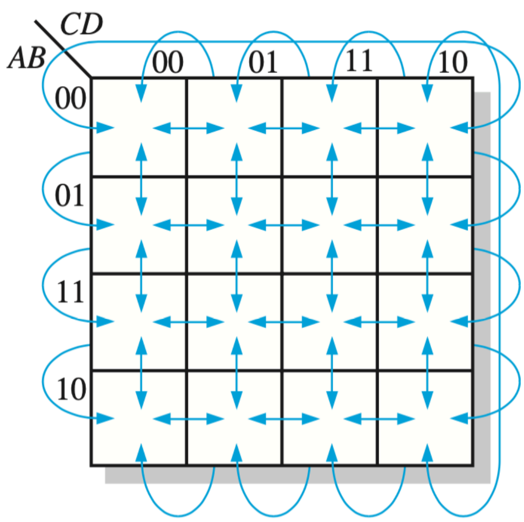


FIGURE 4–27 Adjacent cells on a Karnaugh map are those that differ by only one variable. Arrows point between adjacent cells.

Karnaugh Map SOP Minimization

EXAMPLE 4–23

Map the following standard SOP expression on a Karnaugh map:

$$\overline{A}\,\overline{B}C + \overline{A}B\overline{C} + AB\overline{C} + ABC$$

EXAMPLE 4–24

Map the following standard SOP expression on a Karnaugh map:

$$\overline{A}\,\overline{B}CD + \overline{A}B\overline{C}\,\overline{D} + AB\overline{C}D + ABCD + AB\overline{C}\,\overline{D} + \overline{A}\,\overline{B}\,\overline{C}D + A\overline{B}C\overline{D}$$

Mapping a Nonstandard SOP Expression

EXAMPLE 4–25

Map the following SOP expression on a Karnaugh map: $\overline{A} + A\overline{B} + AB\overline{C}$.

EXAMPLE 4–26

Map the following SOP expression on a Karnaugh map:

$$\overline{B}\,\overline{C} + A\overline{B} + AB\overline{C} + A\overline{B}\,\overline{C}\,\overline{D} + \overline{A}\,\overline{B}\,\overline{C}D + A\overline{B}CD$$

Karnaugh Map Simplification of SOP Expressions

EXAMPLE 4–27

Group the 1s in each of the Karnaugh maps in Figure 4–33.

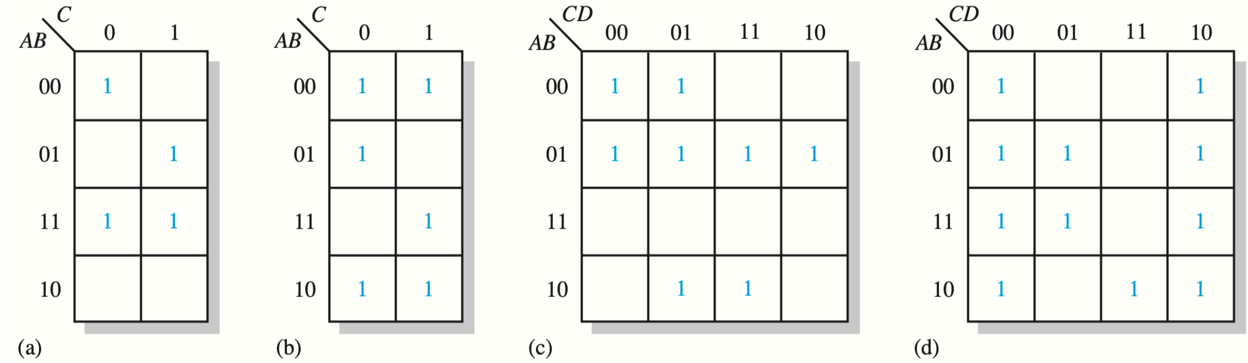


FIGURE 4–33

EXAMPLE 4–28

Determine the product terms for the Karnaugh map in Figure 4–35 and write the resulting minimum SOP expression.

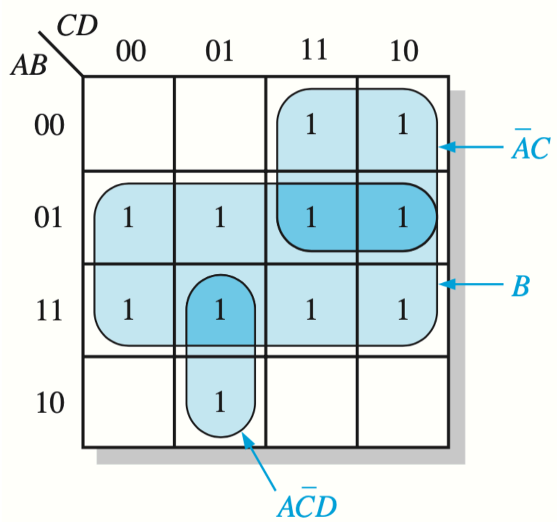


FIGURE 4–35

EXAMPLE 4–30

Use a Karnaugh map to minimize the following standard SOP expression:

$$A\overline{B}C + \overline{A}BC + \overline{A}\,\overline{B}C + \overline{A}\,\overline{B}\,\overline{C} + A\overline{B}\,\overline{C}$$

EXAMPLE 4–31

Use a Karnaugh map to minimize the following SOP expression:

$$\overline{B}\,\overline{C}\,\overline{D} + \overline{A}B\overline{C}\,\overline{D} + AB\overline{C}\,\overline{D} + \overline{A}\,\overline{B}CD + A\overline{B}CD + \overline{A}\,\overline{B}C\overline{D} + \overline{A}BC\overline{D} + ABC\overline{D} + A\overline{B}C\overline{D}$$

Mapping Directly from a Truth Table

$X = \overline{A}\overline{B}\overline{C} + A\overline{B}\overline{C} + AB\overline{C} + ABC$

A	B	C	X
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

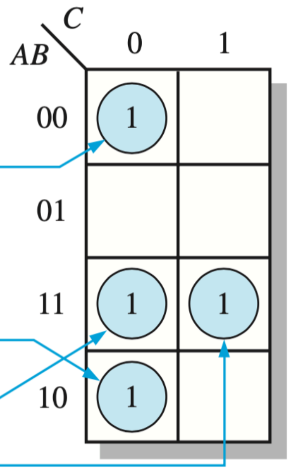


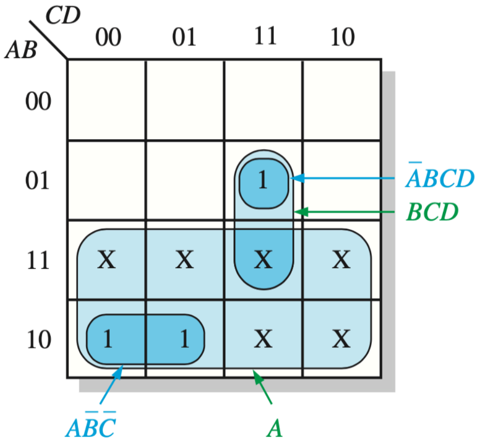
FIGURE 4–39 Example of mapping directly from a truth table to a Karnaugh map.

"Don't Care" Conditions

A	B	C	D	Y
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	X
1	0	1	1	X
1	1	0	0	X
1	1	0	1	X
1	1	1	0	X
1	1	1	1	X

Don't cares

(a) Truth table



(b) Without "don't cares" $Y = A\overline{B}\overline{C} + \overline{A}BCD$
With "don't cares" $Y = A + BCD$

FIGURE 4–40 Example of the use of "don't care" conditions to simplify an expression.

EXAMPLE 4–32

In a 7-segment display, each of the seven segments is activated for various digits. For example, segment *a* is activated for the digits 0, 2, 3, 5, 6, 7, 8, and 9, as illustrated in Figure 4–41. Since each digit can be represented by a BCD code, derive an SOP expression for segment *a* using the variables *ABCD* and then minimize the expression using a Karnaugh map.

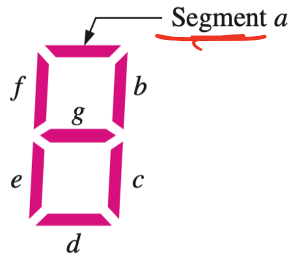
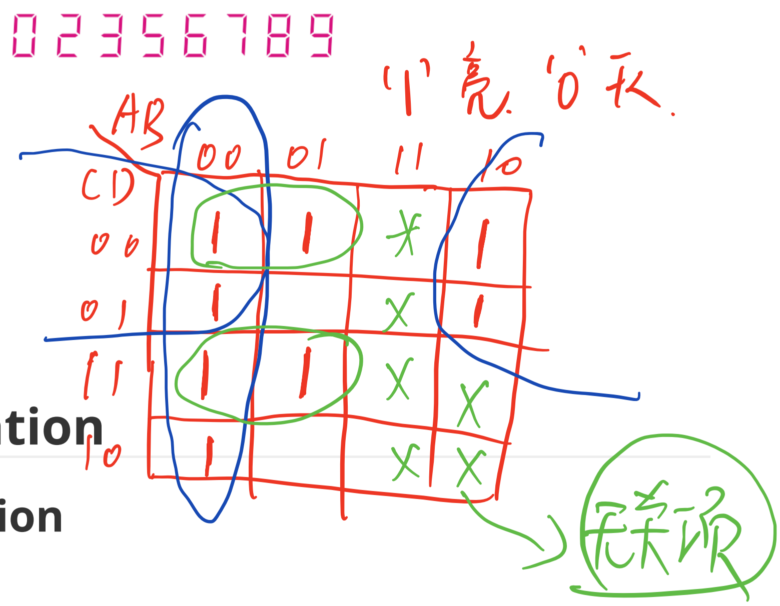
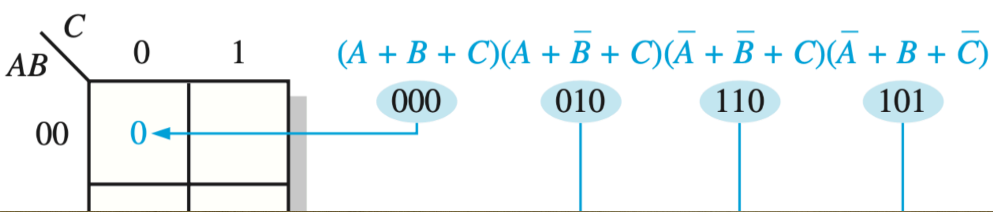


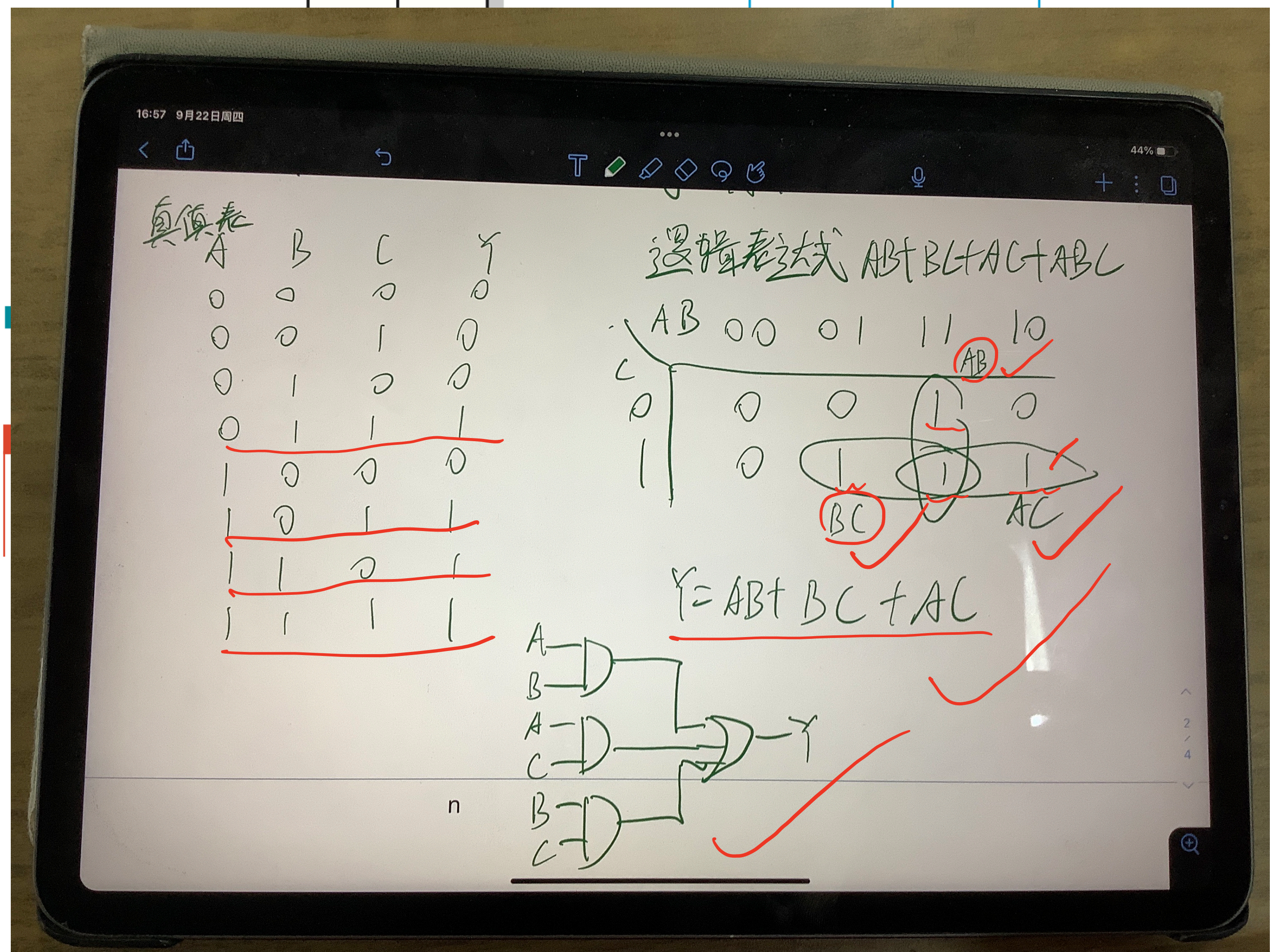
FIGURE 4–41 7-segment display.

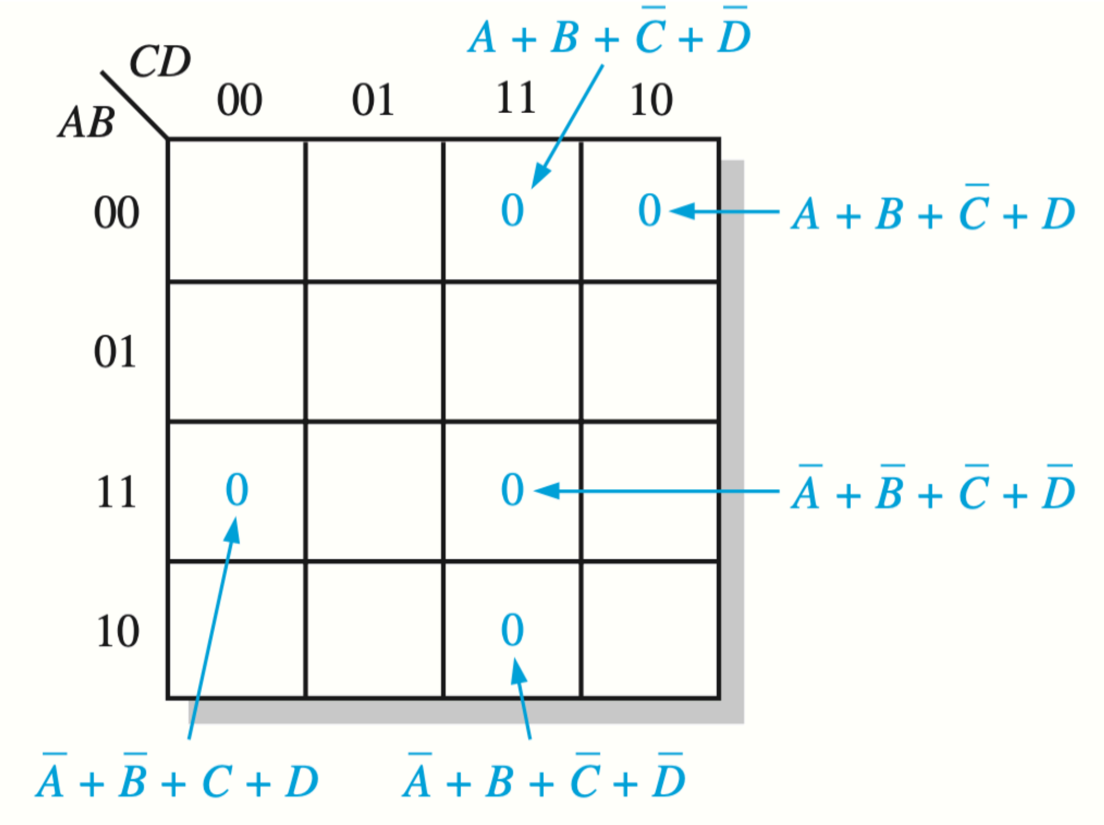


Karnaugh Map POS Minimization

Mapping a Standard POS Expression







EXAMPLE 4–34

Use a Karnaugh map to minimize the following standard POS expression:

$$(A + B + C)(A + B + \overline{C})(A + \overline{B} + C)(A + \overline{B} + \overline{C})(\overline{A} + \overline{B} + C)$$

Also, derive the equivalent SOP expression.

EXAMPLE 4–35

Use a Karnaugh map to minimize the following POS expression:

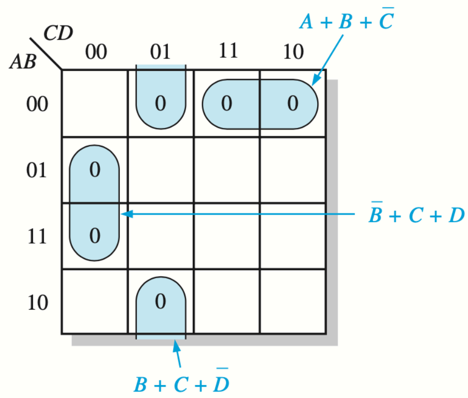
$$(B + C + D)(A + B + \overline{C} + D)(\overline{A} + B + C + \overline{D})(A + \overline{B} + C + D)(\overline{A} + \overline{B} + C + D)$$

Converting Between POS and SOP Using the Karnaugh Map

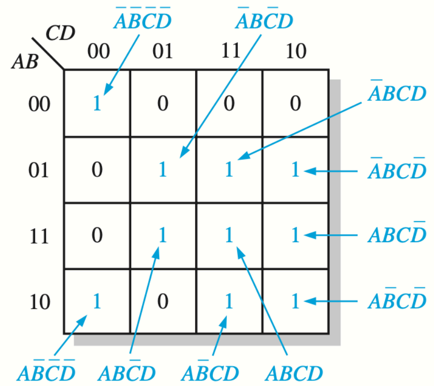
EXAMPLE 4–36

Using a Karnaugh map, convert the following standard POS expression into a minimum POS expression, a standard SOP expression, and a minimum SOP expression.

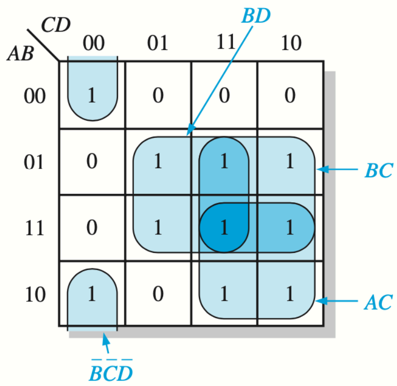
$$(\overline{A} + \overline{B} + C + D)(A + \overline{B} + C + D)(A + B + C + \overline{D})(A + B + \overline{C} + \overline{D})(\overline{A} + B + C + \overline{D})(A + B + \overline{C} + D)$$



(a) Minimum POS: $(A + B + C)(\overline{B} + \overline{C} + D)(B + C + \overline{D})$



(b) Standard SOP:
$\overline{A}\,\overline{B}\,\overline{C}\,\overline{D} + \overline{A}B\overline{C}D + \overline{A}BCD + \overline{A}BC\overline{D} + ABC\overline{D} + A\overline{B}C\overline{D} +$
$A\overline{B}\,\overline{C}\,\overline{D} + AB\overline{C}D + A\overline{B}CD + ABCD$



(c) Minimum SOP: $AC + BC + BD + \overline{B}\,\overline{C}\,\overline{D}$

FIGURE 4–47

SOP

$$Y = ABC + \bar{A}BC + A\bar{B}C + AB\bar{C}$$

$$\overline{\bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + A\bar{B}\bar{C} + \bar{A}B\bar{C}}$$

$$= \overline{\bar{A}\bar{B}\bar{C}}\;\overline{\bar{A}\bar{B}C}\;\overline{A\bar{B}\bar{C}}\;\overline{\bar{A}B\bar{C}}$$

$$\boxed{Y = (A+B+C)(A+B+\bar{C})(\bar{A}+B+C)(A+\bar{B}+C)}$$

POS

$$Y = A\bar{B} + \bar{A}CBD$$
$$+ \bar{A}\bar{B}\bar{C}$$

CD \ AB	00	01	11	10
00	1			1
01	1			1
11		1		1
10				1

$$Y = C\bar{D} + ACD$$
$$+ BC$$

CD \ AB	00	01	11	10
00				
01				
11		1	1	1
10	1	1	1	1

$Y = C\bar{D} + ACD + BC$

$C\bar{D}$
1 0

ACD

$C\bar{D}$

BC.